Hydraulics of bridge, culverts and aqueducts

Introduction

- Whenever
  - a new road has to cross an existing drain or a canal,
  - or sometimes, when a drain has to cross an existing road; a small bridge or a culvert is constructed at the point of crossing.
- **Bridge** is a structure providing **passage (***highway, railway, pedestrian, canal,* pipeline) over **obstacle** *(river, valley, road, railway).*
- *A bridge with span up to 8 m is called a culvert.*

Discharge passing through the bridge openings

- ***Broad Crested Weir formula:*** This formula is applicable as long as the afflux, *i.e. [upstream depth (yu) - downstream depth (yd)] is not less than 1/4Y_d*.(i.e. (yu-yd)> 1/4yd)
- In this formula, the discharge *Q* is dependent only upon *Yu* and is independent of Y_d

$$Q = 1.71 \cdot C_d L \left[y_u + \frac{V_a^2}{2g} \right]^{3/2}$$

  ✓ L= waterway of the bridge, *Yu*= upstream water depth, C_d is the coefficient of discharge,

- The various values of C_d for different types of openings is given as:
  - Narrow Bridge opening with or without floors: 0.94
  - Wide bridge opening with floors: 0.96
  - Wide bridge opening with no bed floors: 0.98

- ***Orifice Formula***: When the downstream depth is more than 80% of the upstream depth *(i.e. the afflux is less than 1/4Y_d)*, the weir formula is not valid, as the performance of the bridge opening gets affected by the downstream depth *(y_d).*
- In such a case, the discharge can be calculated by using the Orifice formula given by:

$$Q = C_0 \sqrt{2g} \cdot L \cdot y_d \cdot \left[h + (1+e) \frac{V_a^2}{2g} \right]^{1/2}$$

Where, h = afflux in meters

Hydraulics of Bridge

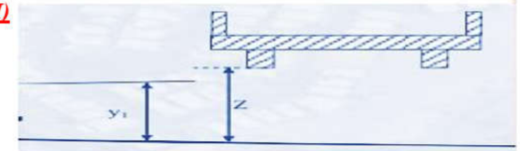
- Introduction

✓ Studying the bridge hydraulics is important to:

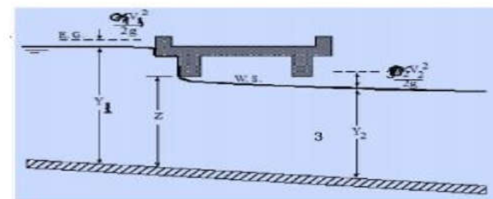
- Check the effect of constructing the bridge on the upstream reach
- Hydraulically inefficient bridge could cause flooding upstream and extremely damaging a large number of properties.
- A properly designed bridge is one that balances the cost of the bridge with concerns of safety, impacts to the environment and regulatory requirements.
- Proper hydraulic analysis and design of bridges is as vital as structural design.

- Types of flow under bridge waterway

A. Low flow case (waterway opening is not submerged) ***(Type I, II or III)***



B. High flow case (waterway opening is submerged) ***Orifice flow***



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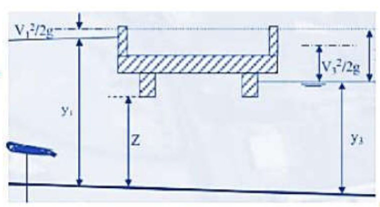
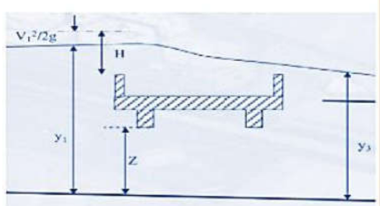
Pressure flow case.



Weir flow case.



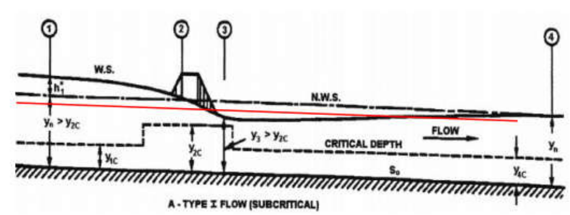
Types of flow in bridge waterway in low flow case

There are three types of flow in bridge waterway at flow low case.

Type I Flow (sub-critical flow)

- *Normal water surface* is everywhere *above critical depth*
- *Actual water surface* is everywhere *above critical depth*
- This type *usually* encountered in practice

Backwater expression for *type-I flow* is obtained by applying the *conservation of energy principle between sections-1 and 4.*



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$$S_0L_{1-4} + y_1 + \frac{\alpha_1 V_1^2}{2g} = y_4 + \frac{\alpha_4 V_4^2}{2g} + h_T \quad \text{............(eq.\#1)}$$

Where, h_T is the total energy loss between sections-1 and 4.

✓ *This method was developed on the basis that the*:

- Channel in the *vicinity of the bridge is essentially straight*.
- Cross-sectional *area of the stream is reasonably uniform* and *the same at sections-1 and 4*.
- *Gradient of the bottom is constant* between sections-1 & 4.
- Applies *only to steady subcritical flow*
- There is *no appreciable scour of the bed in the constriction*
- The *flow is free to contract and expand*.
- Energy loss caused *by constriction* ($h_b = h_T - S_oL_{1-4}$)

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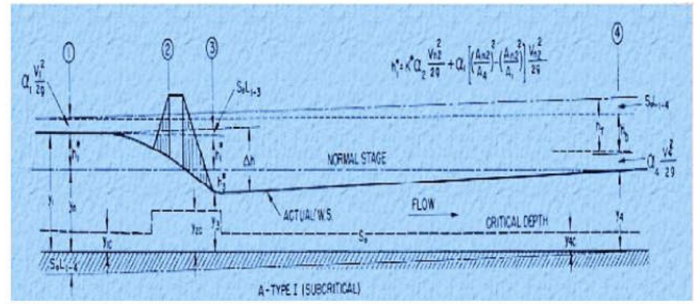

Expression equation #1 can be written as:

$$y_1 - y_4 = \frac{\alpha_4 V_4^2}{2g} - \frac{\alpha_1 V_1^2}{2g} + h_b \quad \text{...............(eq.\#2)}$$

Cont'd...

✓ ***Energy loss*** caused by constriction (h_b) also can be expressed as the product of a loss coefficient, K*, and a velocity head.

$$h_b = K^* \frac{\alpha_2 V^2_{n2}}{2g} \quad \text{...........(eq.\#3)}$$

Where: V_{n2} - is average velocity *in the contracted section* based on the flow area below NWS.

✓ Replacing $(y_1 - y_4)$ with h_1^* and h_b with $K^*\alpha_2 V^2_{n2}/2g$, equation #2 becomes:

$$h^*_1 = K^* \frac{\alpha_2 V^2_{n2}}{2g} + \left[\frac{\alpha_4 V_4^2}{2g} - \frac{\alpha_1 V_1^2}{2g}\right] \quad \text{...........(eq.\#4)}$$

✓ Since the analysis is based on the assumption that the cross *sectional areas at sections-1 and 4 are essentially the same*, α_4 can be replaced by α_1.

✓ Also from the equation of continuity $A_1V_1 = A_4V_4 = A_{n2}V_{n2}$, velocities can be expressed as areas. So the expression for backwater becomes:

Cont'd...

$$h_1^* = K^* \alpha_2 \frac{V^2_{n2}}{2g} + \alpha_1\left[\left(\frac{A_{n2}}{A_4}\right)^2 - \left(\frac{A_{n2}}{A_1}\right)^2\right]\frac{V^2_{n2}}{2g} \quad \text{...........(eq.\#5)}$$

Where,

- $h_1^* = H_1^*$ = total backwater (m)
- K^* = total backwater head loss coefficient
- α_1= velocity head correction coefficient at sections-1 and 4
- α_2= velocity head correction coefficient at constriction
- A_{n2} = gross water area in constriction measured below normal stage (m^2)
- V_{n2} = average velocity in constriction for flow at normal stage or Q/A_{n2} (m/s)
- A_4 = water area at section-4
- A_1 = total water area at section 1, including backwater (m^2)

✓ If piers are present in the constriction, these are ignored in the determination of A_{n2}. (including the area occupied by any piers

Cont'd…

- ✓ The expression represents the difference in kinetic energy between sections-4 and 1.

$$\alpha_1\left[\left(\frac{A_{n2}}{A_4}\right)^2 - \left(\frac{A_{n2}}{A_1}\right)^2\right]\frac{V^2_{n2}}{2g}$$

- ✓ The backwater coefficient, K* can be:

$$K^* = \frac{h_1^*}{\alpha_2 V^2_{n2}/2g} - \frac{\alpha_1}{\alpha_2}\left[\left(\frac{A_{n2}}{A_4}\right)^2 - \left(\frac{A_{n2}}{A_1}\right)^2\right] \quad \text{............(eq.\#6)}$$

Type-II flow (water surface passes through critical depth)

- ✓ There are at least two variations of type II flow which will be described here under types IIA and IIB.

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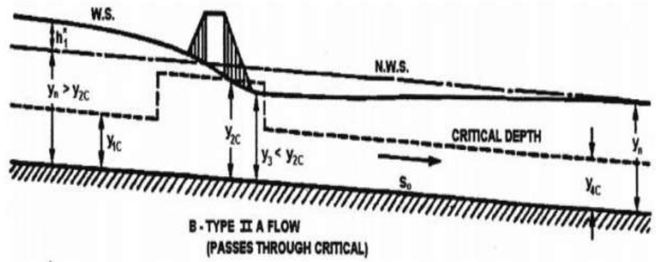
Type-IIA Flow:



- *Normal water surface* is everywhere *above critical depth.*
- *Actual water surface passes* through *critical depth in the constriction.*

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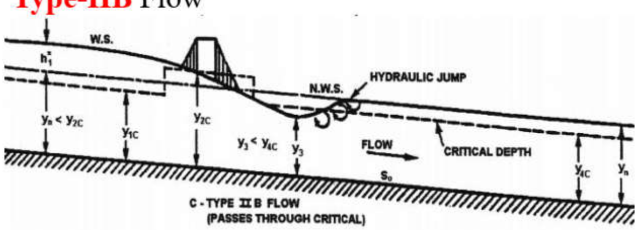
Type-IIB Flow



- *Normal water surface* is *above critical depth* at the upstream and at the downstream (section-4).
- Both *actual water surface and normal water surface* are *below critical depth* at the constriction and passes through critical depth in the constriction.

Cont'd…

- There is *poor hydraulic jump before it returns to normal depth.*
- *Once critical depth is reached, the water surface upstream from* the constriction is no longer influenced by conditions downstream.
- This is true even though the water surface may dip below critical depth, $Y_{2c,}$ in the constriction and then return to subcritical flow as in Type-IIA.

Cont'd…

- Type-IIB flow is similar *except the water surface not only dips below Y_{2c} but also Y_{4c} downstream from the constriction.*
- Both types of flow are subject to the same analysis since the criterion here is that the flow passes through critical depth.
- *A backwater expression* applicable to both types-IIA and IIB flow has been developed by equating the *total energy surface between section-1 and the point at which the water surface passes through critical stage* in the constriction.

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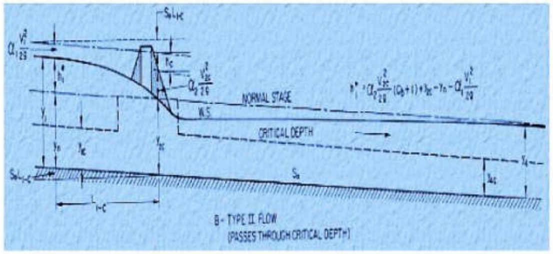

$$S_0L_{1-c} + y_n + h_1^* + \alpha_1\frac{V_1^2}{2g} = Y_{2c} + \alpha_2\frac{V^2_{2c}}{2g} + h_T \quad \text{............(eq.\#7)}$$

- *Energy loss* caused by constriction (h_c) can be expressed as

$h_c = h_T - S_oL_{1-c} = c_b\alpha_2(V^2_{2c})/(2g)$

$$y_n + h_1^* - Y_{2c} = \alpha_2\frac{V^2_{2c}}{2g} + C_b\alpha_2\frac{V^2_{2c}}{2g} - \frac{\alpha_1V_1^2}{2g} \quad \text{............(eq.\#8)}$$

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Cont'd…

✓ Solving for backwater:

$$h_1^* = \frac{\alpha_2V^2_{2c}}{2g}(C_b + 1) + Y_{2c} - y_n - \frac{\alpha_1V_1^2}{2g}, \quad \text{............(eq.\#9)}$$

Where,

- h_1^* = total bridge backwater (m)
- Y_n = normal flow depth (m) (model)
- Y = normal flow depth or $[A_{n2}/b]$ (m) (prototype)
- Y_{2c} = critical depth in constriction or $[(Q^2/(b^2g))^{1/3}]$ (m)
- V_{2c} = critical velocity in constriction or $[Q/(Y_{2c}*b)]$ (m/s)
- V_1 = velocity at section 1 or $[Q/A_1]$ (m/s)
- α_1, α_2 = velocity head correction coefficients at section-1 and in the constriction respectively
- C_b = backwater coefficient for type-II flow (constriction loss only) should the backwater coefficient be desired

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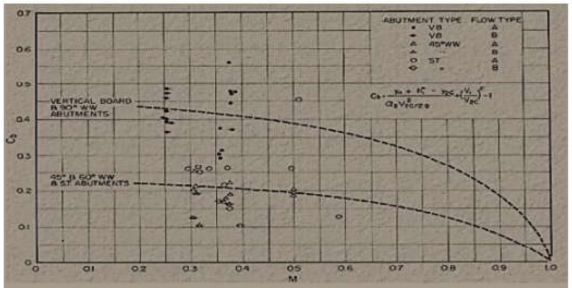
Cont'd…

$$C_b = \frac{h_1^* + y_n - Y_{2c}}{\alpha_2V^2_{2c}/2g} + \frac{\alpha_1}{\alpha_2}\left(\frac{V_1}{V_{2c}}\right)^2 - 1 \quad \text{..............(eq.\#10)}$$



Backwater coefficient curve for type II flow.

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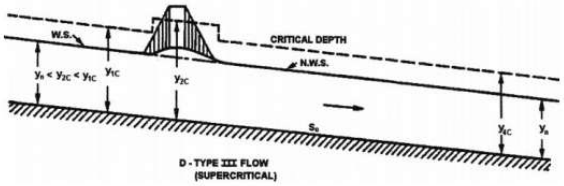
Type-III Flow



- In the type-III flow, figure above the *normal water surface is everywhere below critical depth and flow throughout is supercritical.*
- *This is an rare case requiring a steep gradient but such conditions do exist, particularly in mountainous regions.*
- Theoretically *backwater should not occur for this type, since the flow throughout is supercritical.*

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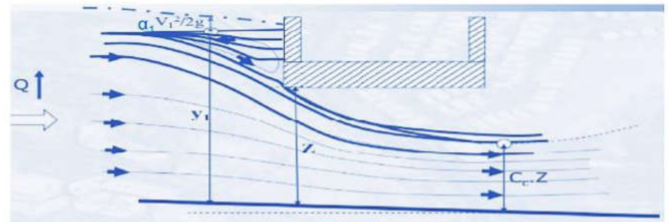
B) High flow case (waterway opening is submerged)

Orifice flow (sluice gate pressure flow)



Cc = 0.5

$$y_1 + \frac{\alpha_1 V_1^2}{2g} = y_{vena} + \frac{V_{vena}^2}{2g} + H_L$$

$$V_{vena} = C_v\sqrt{2g(y_1 - y_{vena} + \frac{\alpha_1 V_1^2}{2g})}$$

$$V_{vena} = C_v\sqrt{2g(y_1 - C_c.Z + \frac{\alpha_1 V_1^2}{2g})}$$

$$V_{vena} = C_v\sqrt{2g(y_1 - 0.5Z + \frac{\alpha_1 V_1^2}{2g})}$$

$$q = y_{vena}.C_v\sqrt{2g(y_1 - 0.5Z + \frac{\alpha_1 V_1^2}{2g})}$$

$$q = C_c Z.C_v\sqrt{2g(y_1 - 0.5Z + \frac{\alpha_1 V_1^2}{2g})}$$

$$q = C_d Z\sqrt{2g(y_1 - 0.5Z + \frac{\alpha_1 V_1^2}{2g})}$$

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Cont'd…

$$Q = C_{d\_orf}.A_o\sqrt{2g(y_1 + \frac{\alpha_1 V_1^2}{2g} - \frac{z}{2})}$$

$$Q = C_{d\_orf}.A_o\sqrt{2g(H)}$$

Where,

- $C_{d\text{-orifice}}$ = is the coefficient of discharge, it ranges from 0.35 – 0.5
- A_o = is the net area of the bridge opening, $A_o = Z*b_{net}$
- Z = is the bridge low chord height
- b_{net} = clear span width of bridge opening.

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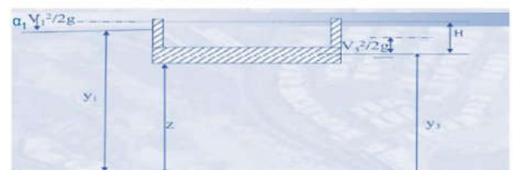
Pressurized Flow



$$Q = C_{d\_pr}.A_0\sqrt{2gH}$$

$$H = Y_1 + \alpha_1\frac{V_1^2}{2g} - Y_3$$

- ✓ Typical values for the discharge coefficient C_d range from 0.7 to 0.9.
- ✓ A value of 0.8 is used for most bridges.

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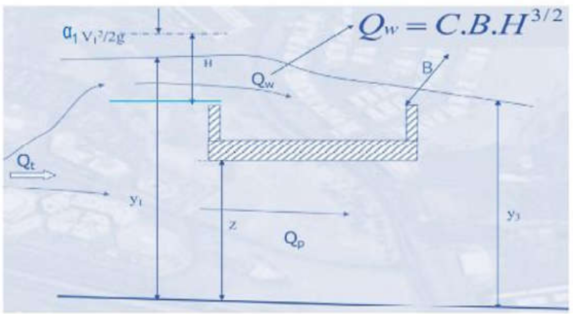
Weir flow



$$Q_{pr} = C_{d\_pr}.A_2\sqrt{2g(y_1 + \frac{\alpha_1 V_1^2}{2g} - y_3)}$$

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Hydraulics of Culverts

Culverts

- Culverts are conveyance structures which carry water under canal, roads, highways etc.
- It consists of a pipe barrel which can be circular or rectangular, and an entrance and exit.
- Flow in a culvert can be either free flow (open channel) or pipe flow.

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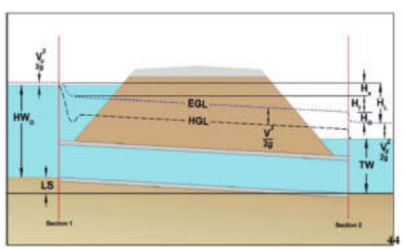
- ✓ Culvert designed hydraulically to increase hydraulic capacity and used to convey surface runoff through embankments.
- ✓ Common culvert diameters are 600mm and 900mm.
- ✓ Cross culverts smaller than 600mm in diameter should not be installed as they are very difficult to clean or maintenance (ERA, 2011).
- ✓ During a storm event a culvert may operate under inlet control, outlet control or both.



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Classification of flow types at culvert

- ✓ A culvert barrel may flow full or partly full over all of its length.
- ✓ The hydraulic condition in a culvert flowing full is called *pressure flow*
  - One condition which can create pressure flow in a culvert is the back pressure caused by a high downstream water surface elevation.
- ✓ *Partly full* (free surface) flow or open channel flow may be categorized as subcritical, critical, or supercritical.
- ✓ A determination of the appropriate flow regime is accomplished by evaluating the dimensionless number called the Froude number.

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- ✓ When $Fr > 1.0$, the flow is supercritical and is characterized as rapid
- ✓ When $Fr < 1.0$, the flow is subcritical and is characterized as tranquil (gradual)
- ✓ If $Fr = 1.0$, the flow is defined as critical
- ✓ $F_r = \frac{v}{\sqrt{gy}}$
- ✓ For steep culvert flowing partly full, critical depth occurs at the culvert inlet, subcritical flow exists in the upstream channel, and supercritical flow exists in the culvert barrel.

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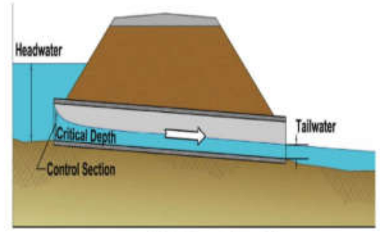
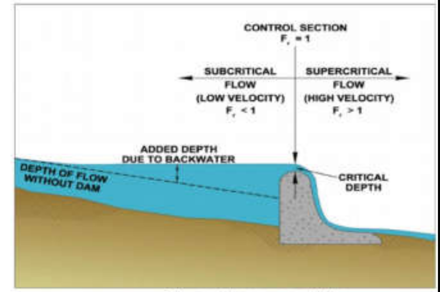


Typical inlet control flow section.



Flow conditions over a small dam.

Culvert flow controls (inlet and outlet control)

- ✓ **Types of flow control**
  - – Control section is the location where there is a unique relationship between the flow rate and upstream water surface elevation.
  - – The characterization of pressure, subcritical and supercritical flow regimes played an important role in determining the location of the control section.

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Inlet control

- ✓ Inlet control occurs when the culvert barrel is capable of conveying more flow than the inlet will accept.
- ✓ The inlet control section is located just inside the entrance.
- ✓ Critical depth occurs at or near this location, and the flow regime immediately downstream is supercritical.
- ✓ Downstream of the inlet control section do not affect the culvert capacity.
- ✓ The upstream water surface elevation and the inlet geometry are the major flow controls.

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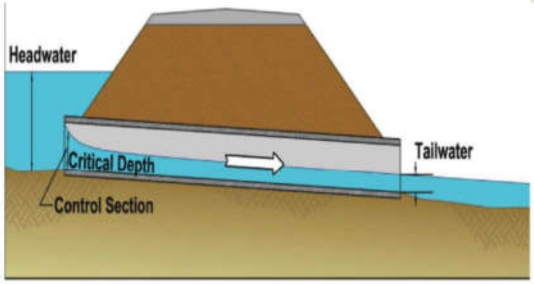

Typical inlet control flow section.

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Outlet control

- ✓ Occurs when the culvert barrel is *not capable of conveying* as much flow as the inlet opening will accept.
- ✓ The control section for outlet control flow in a culvert is located at the barrel exit or further downstream.
- ✓ Either subcritical or pressure flow exists in the culvert barrel under these conditions.
- ✓ All of the geometric and hydraulic characteristics of the culvert play a great role in determining its capacity.

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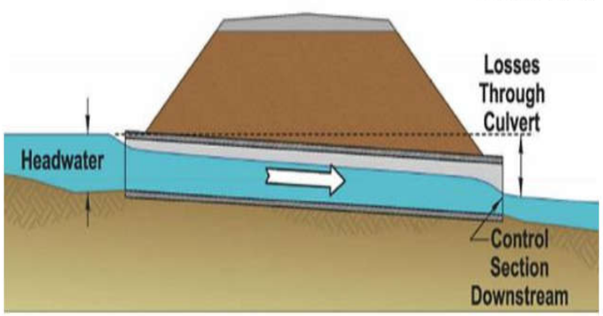


Typical outlet control flow conditions.

Factors influencing culvert performance

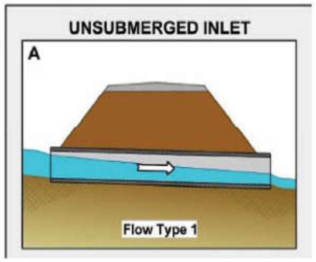
Factor	Inlet control	Outlet control
Headwater elevation	X	X
Inlet area	X	X
Inlet edge configuration	X	X
Inlet shape	X	X
Barrel roughness		X
Barrel area		X
Barrel shape		X
Barrel length		X
Barrel slope	*(to small extent or can be neglected)	X
Tail-water elevation		X

Flow types

Flow type I: characteristics of the flow type-I are:

- Un-submerged inlet & free flow outlet
- Critical depth at inlet
- Partially full flow, steep slope
- Supercritical flow at barrel
- Inlet control.

$\frac{Y_1}{D} < 1.2 \quad and \quad \frac{Y_4}{Y_c} < 1$



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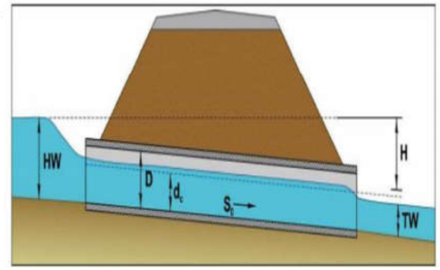
Flow type II: characteristics of the flow type-II are:

- Un-submerged inlet with low tail water
- Critical depth at outlet
- Partially full flow
- Mild slope
- Subcritical flow at barrel
- Outlet control

$\frac{Y_1}{D} < 1.2 \; and \; \frac{Y_4}{Y_c} < 1$



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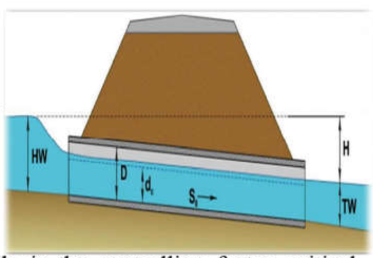
Flow type III: characteristics of the flow type-III are:

- Un-submerged inlet with tail water backing up in to outlet
- Partially full flow
- Mild slope
- Subcritical flow at barrel
- Outlet control

$\frac{Y_1}{D} < 1.2 \; and \; \frac{Y_4}{Y_c} > 1$



- Existence of GVF profile is the controlling factor, critical depth cannot occur, and the upstream water surface elevation is the function of the tail water elevation.

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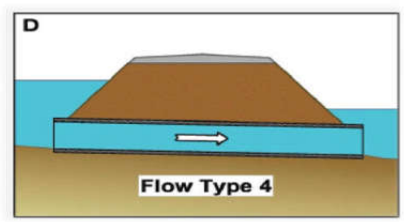
Flow type IV: characteristics of the flow type-IV are:

- Both inlet and outlet fully submerged
- Full flow/pipe flow
- Any slope
- Outlet control

$\frac{Y_1}{D} > 1 \; and \; \frac{Y_4}{D} > 1$



- The flow rate can be estimated directly from the energy equation. The tail water depth is assumed to be critical depth near the culvert outlet or in the downstream channel.

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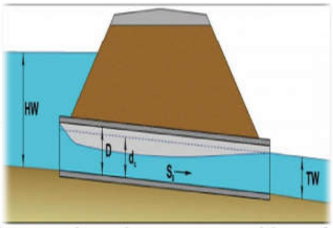
Flow type V: characteristics of the flow type-V are:

- Submerged inlet with low tail water
- Partially full flow
- Any slope
- Short length of culvert
- Orifice flow control at inlet.



$\frac{Y_1}{D} > 1.2 \quad and \quad \frac{Y_4}{Y_c} \leq 1$

- An orifice is an opening, submerged on the upstream side and flowing freely on the downstream side.

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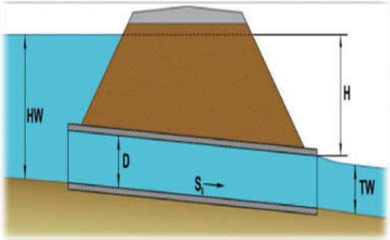
Flow type VI: characteristics of the flow type-VI are:

- Submerged inlet with low tail water
- Culvert flows full /pipe flow
- Outlet not submerged
- Any slope
- Long length of culvert
- Outlet control

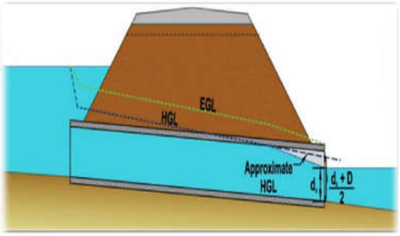


$\frac{Y_1}{D} > 1.2 \; and \; \frac{Y_4}{D} \leq 1$

Cont'd…

Flow type VII: characteristics of the flow type-VII are

- Submerged inlet with low tail water
- Partially full flow
- Outlet not submerged
- Steep slope
- Outlet control



$\frac{Y_1}{D} > 1.2 \quad and \quad \frac{Y_4}{D} \leq 1$

Rip rap erosion protection was required below this culvert. If Fr <3 and Q < 12m3/s, design a riprap basin for each barrel).

A simple energy dissipater at a culvert outlet. (If Fr > 3, design a stilling basin)

The outlet velocities of this culvert are almost 4.5 m/sec. Erosion damage is minimal.

Important formula

Manning's Equation:

$Q = (1/n) * A * (R)^{2/3} * (S)^{1/2}$

Q = Discharge (m3)

n = Hydraulic roughness coefficient

A = Area of the pipe (m 2)

R = Hydraulic radius (m)

S = Slope of the culvert

- **Weir Equation (unsubmerged Inlet):-**

$Q = C_w (L_w) h^{3/2}$

Where:

Q = discharge (m3/s)

C_w = dimensionless weir coefficient (2.2 to 3.1 are typical values for a broad-crested weir)

L_w = the length of the weir perpendicular to the flow (m)

h = is the energy head (depth + velocity head) of the approaching flow above the crest of the spillway. ***If the weir is substantially submerged by tailwater, a reduction factor is usually applied.***

Cont'd…

- **Orifice Equation (Submerged Inlet):**

$Q = C_o A_o (2g\, h_o)^{1/2}$

Where:

Q = discharge (m3/s)

C_o = dimensionless orifice coefficient (0.62 is a typical value for a sharp-edged orifice)

A_o = the area of the orifice perpendicular to the flow (m2)

g = gravitational constant of 9.81 m/sec 2

h_o = the orifice energy head (depth + velocity head) of the approaching flow from inlet centroid (m)

Cont'd…

- ***Energy Grade Line:-*** represents the total energy at any point along the culvert barrel. Equating the total energy at sections 1 and 2, upstream and downstream of the culvert barrel in Figure below, the following relationship results:

$Hwo + SL + (V_u^2/2g) = TW + (V_d^2/2g) + H_L$

Where:

HWo = headwater depth above the outlet invert, m

Vu = approach velocity, m/s

TW = tailwater depth above the outlet invert, m

V_d = downstream velocity, m/s

HL = sum of all losses

L = length of culvert

S = slope of culvert.

Cont'd…

Head Loss through Submerged Tube :(Barrel Losses)

$h_L = he + hf + ho$

Where:

h_L = the total head loss (m)

he = entrance loss (m)

hf = friction loss through the pipe (m)

ho = exit loss (m)

$he = ke (V^2 / 2g)$

Where:

ke = Entrance loss coefficient

V = the flow velocity just inside the barrel inlet (m/ sec)

g = gravitational constant of 9.81m / sec 2

Cont'd…

$hf = (V^2 n^2)(L) / (R^{4/3})$

Where:

- hf = friction loss through the culvert
- V = average barrel velocity, m/s
- L = length of pipe
- A = area of the pipe
- R = hydraulic radius ($R = A / Wp$ - area divided by the wetted perimeter)
- n = Hydraulic roughness coefficient (barrel roughness)

$ho = ko\,[(V^2/2g) - (V_d^2/2g)]$, $ko = 1$

Where:

V_d = channel velocity downstream of the culvert, m/s (usually neglected,).

$ho = hv = V^2/2g$

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Cont'd…

Total Loss:

$h_L = [\,1 + ke + (19.62\, n^2 L) / R^{4/3}\,]\, V^2 / 2g$

Critical Depth:

$Q^2 / g = A_c^3 / Tc$…………for circular shape

Where:

- Q = discharge
- g = the gravitational constant
- Ac = cross sectional area at critical flow
- Tc = top width of water surface at critical flow

For rectangular sections, critical depth can be directly computed as:

$Yc = (q^2 / g)1/3$ *Where:*

- q = discharge per unit width(Q/B)
- g = the gravitational constant

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